You may only reply to two problems in this exam. If the examinee has answered all three problems, the best answer will be disregarded.

Problem 1

a) *(5 points)* Five friends, Kate, Mary, Anne, Eve and Heidi have their own mobile phones and send each other text messages (SMSs). Kate and Heidi have a T-line subscriber connection, Mary and Eve have a Videa connection and Anne has a Jupera connection. The cost of SMSs from one service provider's connection to another are as follows:

Cost of SMSs between different service providers' connections				
		Receiver		
		T-line	Videa	Jupera
Sender	T-line	1 euros	5 euros	4 euros
	Videa	8 euros	3 euros	4 euros
	Jupera	7 euros	6 euros	2 euros

Anne wants to invite all her friends to a party. She can do that either by sending an SMS to all her friends, or by sending an SMS to some of her friends and asking them to forward the message (both announcements can be made in the same SMS). How should she proceed to make the total cost to all the friends of relaying the party invitation as low as possible?

b) *(10 points)* A new friend, Tina, is joining their set. To which service provider's connection should she subscribe in order to make the cost of a party invitation from her to the whole set of friends as low as possible? It is possible to forward the invitation as in item a).

c) *(10 points)* Tina's father gives her a Jupera subscription. Now, all the service providers announce that they will change their SMS prices, but the new prices will not be announced until next week. Tina will send her invitations after the new prices have become valid. She asks you for instructions on how she should send her invitations. The instructions should be so clear that Tina doesn't have to figure anything out herself, just follow the instructions. The instructions may include comparisons with alternative deductions, or calculations like "if (D+E)<(A+B), then send the invitation to Eve and ask her to send it on to Kate," but do not rely on Tina's own powers of deduction outside the instructions. In your instructions, use the annotation A–I for the SMS prices, as in the table below. Further, assume that sending a message to the same service provider's connection is always cheaper than sending a message to another service provider's connection (so the prices A, E and I are cheaper than the others). Write the instructions for Tina on how she should choose the cheapest alternative for sending her invitations.

Cost of SMSs between different service providers' connections				
		Receiver		
		T-line	Videa	Jupera
Sender	T-line	A euros	B euros	C euros
	Videa	S euros	E euros	F euros
	Jupera	G euros	H euros	I euros

Problem 2

Implement a program that outputs a comparison of mobile phone subscriber connections. As input, the program is given the daytime and the evening/night-time phone rates (cents/minute) for each service provider, as well as the monthly rates (euros/month). Assume that all input data consists of integers. The program also requires the number of connection types to be compared, and the amount of daytime calls in relation to all calls (%).

On the basis of this data, the program outputs a table showing the cheapest connection type for different amounts of calls per month. The length of the calling time to be examined should be easy to change by just changing a variable; in the following example it is 0–300 minutes/month. Please note that, in this problem, we are only examining the total time used for phoning, and the number of calls has no effect on the result.

The following is a sample run (the underlined entries are made by the user). Your answer does not have to be formulated exactly like the example, e.g. the connection type data can be given each on a separate line.

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How many connection types will be compared:  3
Give rates for connection 1:  daytime (c) night-time (c) monthly (e):  33 12 2
Give rates for connection 2:  daytime (c) night-time (c) monthly (e):  18 14 10
Give rates for connection 3:  daytime (c) night-time (c) monthly (e):  18 18 4
Give percentage of daytime calls (0..100):  35

With the call amount 0 min/month connection 1 is the cheapest
With the call amount 30 min/month connection 1 is the cheapest
With the call amount 60 min/month connection 1 is the cheapest
With the call amount 90 min/month connection 1 is the cheapest
With the call amount 120 min/month connection 1 is the cheapest
With the call amount 150 min/month connection 1 is the cheapest
With the call amount 180 min/month connection 3 is the cheapest
With the call amount 210 min/month connection 3 is the cheapest
With the call amount 240 min/month connection 2 is the cheapest
With the call amount 270 min/month connection 2 is the cheapest
With the call amount 300 min/month connection 2 is the cheapest
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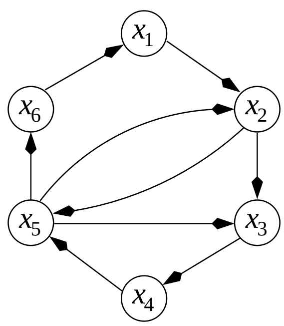
Problem 3

Please read the material carefully. Among the problems, there are some in which the material can be applied directly, and some in which conclusions should be based on the material. The problems are not necessarily in the order of their degree of difficulty.

A suburb will be built near a Finnish town that shall remain anonymous. The architects have finished their plans, and the road network has already been completed. One hospital will be built in the area, and from this hospital, the ambulance on duty must reach any house in the region as quickly as possible in case of emergency. The following will present a method with which to find the optimal location for the hospital.

Since none of the sites in the suburb have any buildings on them yet, the hospital and the apartment buildings can be placed at will. The routes between different sites form a graph with vertices, and there are arcs (arrows) of various lengths between these vertices. The vertices represent sites and the lengths of individual arcs represent the distance between two sites. In the illustrations, the vertices are shown by circles.

Since there is only one-way traffic on some of the streets, the distance between two sites is not necessarily the same in both directions when driving an ambulance. Figure 1 shows a sample graph of six sites, with only one two-way street. In the figure, the lengths of the arcs have not been mentioned, but they are all assumed to be one unit long. This is also assumed for the other illustrations and problems here, but is not generally true.



	x_1	x_2	x_3	x_4	x_5	x_6	$s_u(x_i)$
x_1	0	1	2	3	2	3	3
x_2	3	0	1	2	1	2	3
x_3	4	3	0	1	2	3	4
x_4	3	2	2	0	1	2	3
x_5	2	1	1	2	0	1	**2**
x_6	1	2	3	4	3	0	4
$s_s(x_i)$	4	**3**	**3**	4	**3**	**3**	

Figure 1: A sample graph of six sites and the distances between the sites (vertices).

In the table in Figure 1, the shortest distance between each site and the other sites has been calculated. The vertices are ordered in rows and columns in the table. The vertices for rows represent the sites from which we start, and the vertices for columns represent the site to which we are going.

The distance of the shortest route between vertex x_i and vertex x_j is notated as $e(x_i, x_j)$. In Figure 1, from vertex x_4 to vertex x_6, for example, there is a route that is 2 units long, i.e. $e(x_4, x_6) = 2$. This can be interpreted so that when going from site 4 to site 6, you have to go 2 units of distance. Please note that $e(x_6, x_4) = 4$. The distance between two nodes does not always differ; the distance between vertices x_2 and x_5, for example, is the same in both directions.

We now define two *separation numbers* for each vertex x_i as follows:

$$s_u(x_i) = \max \{e(x_i, x_j) \mid x_j \text{ is a graph vertex}\}$$

and

$$s_s(x_i) = \max \{e(x_j, x_i) \mid x_j \text{ is a graph vertex}\}.$$

Of the maximums above, the first one is derived from a set of elements consisting of the distances from vertex x_i to each other vertex x_j. In the example, we can go from vertex x_1 to vertices x_1, x_2, x_3, x_4, x_5 and vertex x_6. Their distances are 0, 1, 2, 3, 2 and 3 respectively. The maximum is 3, for the distance from vertex x_1 to either vertex x_4 or to vertex x_6.

The number $s_u(x_i)$ is called the *outseparation* of vertex x_i and the number $s_s(x_i)$ is called the *inseparation* of vertex x_i. Please note that the number $s_u(x_i)$ is the largest number in row x_i, and accordingly, the number $s_s(x_i)$ is the largest number of the column x_i. In Figure 1, the outseparations $s_u(x_i)$ and the inseparations $s_s(x_i)$ are shown in the matrix.

The smallest outseparation of the graph is called the graph's *outradius* and is notated r_u, i.e.

$$r_u = \min \{s_u(x_i) \mid x_i \text{ is a graph vertex}\}.$$

Accordingly, the smallest inseparation of the graph is called the graph's *inradius* and is notated r_s, i.e.

$$r_s = \min \{s_s(x_i) \mid x_i \text{ is a graph vertex}\}.$$

The vertex x_u for which the following is true:

$$s_u(x_u) = r_u,$$

is called the graph's *outcentre*. Accordingly, the vertex x_s for which the following is true:

$$s_s(x_s) = r_s,$$

is called the graph's *incentre*.

One graph may have several out- and incentres. In such cases, they form outcentral or incentral sets.

The graph in Figure 1, for example, has only one outcentre (vertex x_5). The graph's outradius is 2. However, there are several incentres, forming an incentral set $\{x_2, x_3, x_5, x_6\}$. The inradius is 3.

With the help of the presentation above, we can run an elementary search for the location of the hospital on such a site that the hospital can send an ambulance as quickly as possible to the location furthest away from the hospital. However, sometimes the transportation of the patient to and from the hospital is more important than giving first aid quickly. For this purpose, let us define a combined *in-outseparation* of the vertex x_i:

$$s_{su}(x_i) = \max \{e(x_i, x_j) + e(x_j, x_i) \mid x_j \text{ is a graph vertex}\}.$$

The *in-outradius* is the smallest in-outseparation of the graph, i.e.

$$r_{su} = \min \{s_{su}(x_i) \mid x_i \text{ is a graph vertex}\}.$$

The *in-outcentre* is specified in the same way as the earlier centres; vertex x_{su}, for which the following holds

$$s_{su}(x_{su}) = r_{su},$$

is called the in-outcentre of the graph. There can be several in-outcentres. In the graph in Figure 1, the in-outcentre is the vertex x_5, and its in-outradius is 4.

Problems

Please answer the following seven problems, writing your answers carefully and legibly on your paper so that they cannot be mixed up. If, for example, you are required to give the out- and inseparations of a vertex, do not write only two numbers, but also write which one is the inseparation and which one the outseparation.

1. *(2 points)* Does the graph in **Figure 1** have any other in-outcentres than vertex x_5? Give arguments for your answer. If your answer is yes, write down the other in-outcentres.

2. In **Figure 2**, the outseparation of vertex x_1 is $s_u(x_1) = 2$ and its inseparation is $s_s(x_1) = 1$.

   (a) *(1 point)* What are the outseparation and inseparation of vertex x_2?

   (b) *(1 point)* What are the outseparation and inseparation of vertex x_3?

3. *(2 points)* Let us look at the graph in **Figure 2**. List all the outcentres and incentres of the graph.

4. In **Figure 3**, all the distances have been calculated with a few exceptions.

   (a) *(1 point)* What is the shortest distance from vertex x_5 to vertex x_9?

   (b) *(1 point)* What is the shortest distance from vertex x_8 to vertex x_6?

5. Let us look at the graph in **Figure 3**.

   (a) *(2 points)* What are the outseparation $s_u(x_5)$ and the outseparation $s_u(x_8)$?

   (b) *(2 points)* What are the inseparation $s_s(x_6)$ and the inseparation $s_s(x_9)$?

   (c) *(2 points)* What are the outradius and the inradius of the graph?

   (d) *(2 points)* List all the incentres and outcentres of the graph.

   (e) *(2 points)* List all the in-outcentres of the graph.

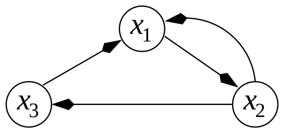
   (f) *(1 point)* What is the in-outradius of the graph?

6. One of the definitions in the written material can be interpreted to tell us the vertex, from which the longest distance to each other vertex by the shortest route is as short as possible. (There can be several such vertices.)

   (a) *(2 points)* Which concept definition are we looking for?

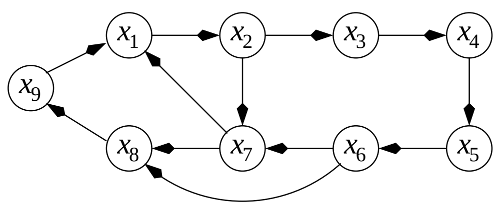
   (b) *(2 points)* From which vertex or vertices is the longest distance to each other vertex by the shortest route as **long** as possible? (In the graph in **Figure 3**)

7. *(2 points)* Let us assume that the definitions for the outseparations and inseparations in the material are changed so that we use the mean distance instead of the maximum. Which vertices then become outcentres and incentres in the graph in **Figure 1**?



	x_1	x_2	x_3	$s_u(x_i)$
x_1	0	1	2	2
x_2	1	0	1	
x_3	1	2	0	
$s_s(x_i)$	1			

Figure 2: A graph of three sites and a matrix of the distances between vertices.



	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	$s_u(x_i)$
x_1	0	1	2	3	4	5	2	3	4	5
x_2	2	0	1	2	3	4	1	2	3	4
x_3	5	6	0	1	2	3	4	4	5	6
x_4	4	5	6	0	1	2	3	3	4	6
x_5	3	4	5	6	0	1	2	2		
x_6	2	3	4	5	6	0	1	1	2	6
x_7	1	2	3	4	5	6	0	1	2	6
x_8	2	3	4	5	6		4	0	1	
x_9	1	2	3	4	5	6	3	4	0	6
$s_s(x_i)$	5	6	6	6	6		4	4		

Figure 3: A graph of nine sites and a matrix of the distances between vertices.